Letter to the Editor of the Physical Review

Density Matrix in the Theory of the Positron

In the present theory of the electron and the positron, only one sort of them is considered at first, the existence of the other being deduced as necessary consequence of the theory. One can proceed, however, on the reverse way, accepting the existence of both at the beginning and introducing afterwards theoretically possible relations between them. The mathematical formulation of the latter method will be as follows.

The quantized wave functions $\psi_-(x\,k)$ and $\psi_+(x\,k)$ of the electron and the positron satisfy Dirac's equations

$$\left\{\frac{W \pm eV}{c} + \vec{\alpha}\left(\vec{p} \pm \frac{e}{c}\vec{A}\right) + \beta mc\right\}\psi_{\mp} = 0 \qquad (1)$$

respectively, where x denotes position and time and k takes either of the values 1, 2, 3, 4. If we adopt a representation, in which all matrix elements of α's are real and those of β are pure imaginary, the wave functions ψ_+^* and ψ_-^*, which are complex conjugate to ψ_+ and ψ_- respectively, satisfy the same equations (1) for ψ_- and ψ_+ respectively, so that if the relations

$$\psi_- = \psi_+^* \qquad \psi_+ = \psi_-^* \qquad (2)$$

are assumed at an instant for all points, they will remain to hold good forever. These are obviously mathematical expressions of the equivalence of the anti-electron and the positron on the