Letter to the Editor
of the Physical Review

The Density Matrix in the Theory of the Positron

In the usual Theory of the electrons and the positrons, only one sort of them, is considered at first, the existence of the other, ~~the electron for example~~, ~~i.e. the positron~~ being deduced as the necessary consequence of the theory. One can proceed, however, on the reverse way, accepting the existence of both at the beginning and only afterwards introducing ~~possible~~ theoretically possible relations between them. The mathematical formulation of the latter method will be as follows.

The quantized wave functions $\psi_-(x,k)$ and $\psi_+(x,k)$ of the electron and the positron satisfy Dirac's equations

$$\left\{ \frac{W \pm eV}{c} + \vec{\alpha}\left(\vec{p} \pm \frac{e}{c}\vec{A}\right) + \rho mc \right\} \psi_\mp = 0 \qquad (1)$$

respectively, where x denote position and time ~~of the particle~~ and k takes either of the values 1, 2, 3, 4. If we adopt a representation, in which all matrix elements of α's are real and those of ρ are pure imaginary, the wave functions ψ_-^* and ψ_+^*, which are complex conjugate to ψ_- and ψ_+ respectively, satisfy the same equations (1) for ψ_+ and ψ_- respectively, so that if the relations

$$\psi_- = \psi_+^* , \qquad \psi_+^* = \psi_- \qquad (2)$$

are assumed at an instant for all points, they will remain to hold good forever. These are obviously a mathematical expressions of the equivalence of the anti-